Enhancement of superconductivity in dirty films in an external magnetic field

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Thin dirty superconducting films containing magnetic impurities exhibit nontrivial behavior when subjected to an applied magnetic field. This behavior manifests itself in the **enhancement of superconductivity**, which is attributed to the reduction in the exchange scattering rate **due to the polarization of the impurity spins** in the presence of the magnetic field. This effect was theoretically predicted by Kharitonov and Feigelman in Ref. [1] and then observed experimentally in Ref. [2]. In both studies, the authors concentrated on the dependence of the critical temperature on the magnetic field parallel to the film surface, revealing a nonmonotonic behavior characterized by a maximum at finite field values. However, manifestations of this enhancement for other observable physical quantities, as well as the description of the effect in the presence of a magnetic field component perpendicular to the film surface, have not been investigated. To address this gap, we develop a theoretical framework employing Gor'kov's diagrammatic technique for superconductors [3]. Our work extends the theory of Ref. [1] in two directions: (i) we demonstrate that the enhancement is also reflected in an **increase in superfluid density and the spectral energy gap**; (ii) we reveal a **nonmonotonic dependence of the second critical field** (perpendicular to the film surface) on the magnetic field component parallel to the surface.

Model

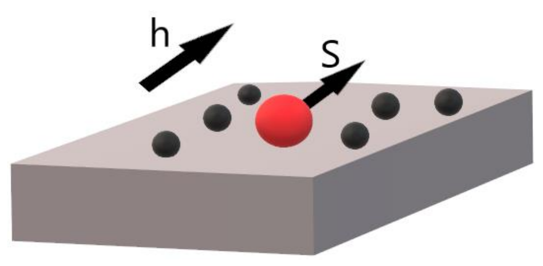


Fig.1: Thin dirty superconducting film containing magnetic impurities with spins $\boldsymbol{S}$ and subjected to the external magnetic field $\boldsymbol{h}$.

Superconducting film with potential and magnetic disorder in the magnetic field:

$$H = H_{BCS} + H_S + H_{pot} + H_{eS}$$

SC subjected to magnetic field

$$H_{BCS} = \int \left\{ \psi_\alpha^\dagger \left(\frac{1}{2m}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)^2 - \varepsilon_F \right) \psi_\alpha - \psi_\alpha^\dagger h \sigma_{\alpha\beta}^z \psi_\alpha + \lambda \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha \right\} d^3\boldsymbol{r}$$

Magnetic impurities

$$H_S = -\sum_a \omega_s S_a^z \qquad \boldsymbol{\omega_s} = \boldsymbol{g_s h} \ \text{–Zeeman energy}$$

Scattering on potential impurities with rate ν_0

$$H_{pot} = \int \left\{ \psi_\alpha^\dagger \sum_a (u_0 \delta_{\alpha\beta}) \delta(\boldsymbol{r} - \boldsymbol{R}_a)\ \psi_\beta \right\} d^3\boldsymbol{r}$$

Scattering on magnetic impurities with rate ν_s

$$H_{eS} = \int \left\{ \psi_\alpha^\dagger \sum_a J(\boldsymbol{S}_a, \boldsymbol{\sigma}_{\alpha\beta}) \delta(\boldsymbol{r} - \boldsymbol{R}_a)\ \psi_\beta \right\} d^3\boldsymbol{r}$$

Superconductor with magnetic impurities

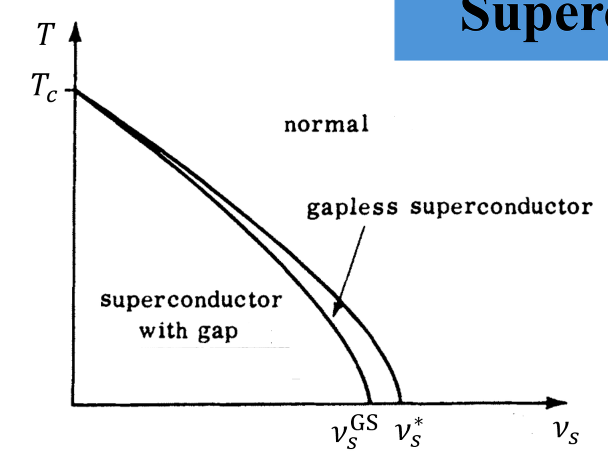


Fig.2: Superconductivity in the presence of magnetic impurities.

- Gapless superconductivity at $\nu_s^* > \nu_s > \nu_s^{GS}$:

$$\nu_s^{GS} = 2\nu_s^* \exp\left(-\frac{\pi}{4}\right), \qquad \varepsilon_g = \Delta_0 \left(1 - \left(\frac{\nu_s}{\Delta_0}\right)^{2/3}\right)^{3/2}$$

Scattering on magnetic impurities leads to the suppression of the SC [4].

Main results of the Abrikosov-Gor'kov's (AG) theory:

- Green's function equation:

$$\frac{\varepsilon_{n0}}{\Delta_0} = u\left(1 - \frac{\nu_s}{\Delta_0\sqrt{1+u^2}}\right), \qquad u = \frac{\varepsilon_n}{\Delta}\ \text{– renormalized energy and order parameter}$$

- Suppression of the critical temperature (with full suppression at ν_s^*):

$$\ln\frac{T_c}{T_{c0}} = \pi T \sum_{\varepsilon_{n0}} \frac{1}{\varepsilon_{n0}} - \frac{1}{\varepsilon_{n0} + \nu_s}, \qquad \nu_s^* = \frac{\Delta_{00}}{2} = \frac{\pi T_{c0}}{2e^C}$$

- Suppression of the superfluid density n_{sc} with the increasing ν_s

General feature: monotonic decrease of all quantities with increasing $\boldsymbol{\nu_s}$

T_c enhancement

Mechanism: polarization of impurity spins by magnetic field (KF theory [1])

→ Reduction of exchange (ES) scattering rate on magnetic disorder → SC enhancement due to AG theory

Neglecting orbital (OE) and paramagnetic effect (PE) of the magnetic field

Critical temperature:

$$\ln\frac{T_{c0}}{T} = \pi T \sum_{\varepsilon_{n0}} \left(\frac{1}{\varepsilon_{n0}} - C_0(\varepsilon_{n0})\right)$$

Equation for Cooperon:

$$\left(|\varepsilon_{n0}| + \nu_{ES}^{(\text{eff})}(\varepsilon_{n0}) + \frac{1}{2}\left(\hat{L}_0 - \nu_\perp(\varepsilon_{n0})\right)\right) C_0 = 1$$

Full ES rate

$$\nu_{ES}^{(\text{eff})}(\varepsilon_{n0}) = \nu_z + \nu_\perp(\varepsilon_{n0})$$

ES rate with spin-flip

$$\nu_\perp(\varepsilon_{n0}) = \nu_\perp T \sum_{|\omega|<|\varepsilon_{n0}|} \frac{\omega_s}{\omega_s^2 + \omega^2}$$

$$\nu_\perp = \nu_s \langle S_z \rangle / S(S+1)$$

ES rate without spin-flip

$$\nu_z = \nu_s \langle S_z^2 \rangle / S(S+1)$$

Nonlocal $\hat{L}_0$ operator

$$\hat{L}_0 C_0(\varepsilon_{n0}) = \nu_\perp T \sum_{|\omega|<|\varepsilon_{n0}|} \frac{2\omega_s}{\omega_s^2 + \omega^2} C_0(\varepsilon_{n0} - \omega)$$

$\boldsymbol{\omega_s}$=0

$$C_0 = \frac{1}{|\varepsilon_{n0}| + \nu_s}, \qquad \nu_z = \frac{\nu_\perp}{2} = \frac{1}{3}\nu_s$$

$\boldsymbol{\omega_s}$=∞

$$C_0 = \frac{1}{|\varepsilon_{n0}| + \nu_\infty}, \qquad \nu_\infty = \nu_z = \frac{S}{S+1}\nu_s$$

OE and PE of the magnetic field

In the absence of OE and PE: monotonic enhancement of SC

Problem		Solution	
Orbital effect: suppression of SC by gradient term in H_{BCS}	→	Magnetic field **parallel** to film surface with small thickness d	$(p_F d)^2 \ll \nu_0 / T_{c0}$
Paramagnetic effect: suppression of SC by Zeeman term in H_{BCS}	→	Presence of high **spin-orbital scattering** rate ν_{so}	$\nu_{so}/\nu_s \gg 1$

$$H_{eS} = \int \left\{ \psi_\alpha^\dagger(\boldsymbol{r}) \sum_a \nu_{\alpha\beta}(\boldsymbol{r} - \boldsymbol{R}_a, \boldsymbol{r}' - \boldsymbol{R}_a)\ \psi_\beta(\boldsymbol{r}') \right\} d^3\boldsymbol{r}\, d^3\boldsymbol{r}',$$

$$\nu_{\alpha\beta}(\boldsymbol{p}, \boldsymbol{p}') = i\left(\nu_{SO}/p_F^2\right)\left([\boldsymbol{p}, \boldsymbol{p}'], \sigma_{\alpha\beta}\right)$$

Equation for Cooperon with OE and PE of $h_\parallel$:

$$\left(|\varepsilon_{n0}| + \nu_{ES}(\varepsilon_{n0}) + \frac{1}{2}\left(\hat{L}_0 - \nu_\perp(\varepsilon_{n0})\right) + \gamma_\parallel\right) C_0 = 1,$$

$$\gamma_\parallel = \gamma_{OE} + \gamma_{PE} \quad \text{– pair-breaking parameter}$$

$$\gamma_{OE} = \frac{2(p_F d)^2 (h_\parallel)^2}{9\nu_0}, \qquad \gamma_{PE} = \frac{3\,(h_\parallel')^2}{2\nu_{SO}} \qquad h_\parallel' = h_\parallel - \nu_S \zeta \langle S_z \rangle / S$$

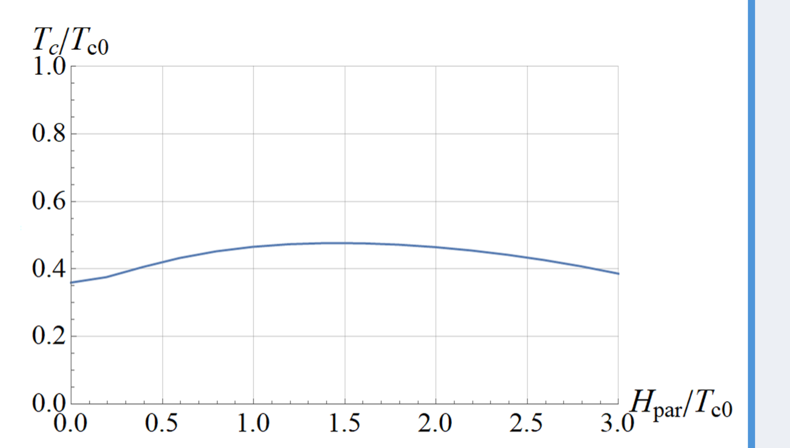


Fig.3: $T_c(H_{par})$ dependence at $\nu_s = 0.7\,T_{c0}$, $\nu_{so} = 10^3 T_{c0}$, $\nu_0 = 10^4 T_{c0}$, $p_F d = 30$, $p_F l = 10$, $\zeta = 5$

Diagrammatic technique

Spin correlation functions

$$L_{-+}(i\omega_m) = \int_0^{1/T} \langle S_-(\tau) S_+(0) \rangle e^{i\omega_m \tau} d\tau = \frac{\langle 2S_z \rangle}{i\omega_m + \omega_s} \qquad L_{+-}(i\omega_m) = \int_0^{1/T} \langle S_+(\tau) S_-(0) \rangle e^{i\omega_m \tau} d\tau = \frac{\langle 2S_z \rangle}{-i\omega_m + \omega_s}$$

$$L_{zz}(i\omega_m) = \int_0^{1/T} \langle S_z(\tau) S_z(0) \rangle e^{i\omega_m \tau} d\tau = \frac{\langle S_z^2 \rangle\, \delta_{\omega_m 0}}{T} \qquad \langle S_z \rangle = \left(S + \frac{1}{2}\right)\coth\left[\left(S + \frac{1}{2}\right)\frac{\omega_s}{T}\right] - \frac{1}{2}\coth\frac{\omega_s}{2T}$$

Our goal: Extend the theory of Ref. [1] to arbitrary temperatures → Gor'kov's diagrammatic technique

Dirty superconductor with magnetic field

$$G_\pm^{q_s}(i\varepsilon_n, p) = \frac{-1}{\Delta_\pm^2 + \xi_p^2 - (i\varepsilon_n \mp h_\parallel' + \boldsymbol{v_p} \cdot \boldsymbol{q_s})^2}\left((i\varepsilon_n \mp h_\parallel' + \boldsymbol{v_p} \cdot \boldsymbol{q_s})\hat{\tau}_0 + \xi_p \hat{\tau}_z - \Delta_\pm \hat{\tau}_+ - \Delta_\pm^* \hat{\tau}_-\right), \qquad \boldsymbol{q_s} = \frac{2e}{c}\boldsymbol{A}$$

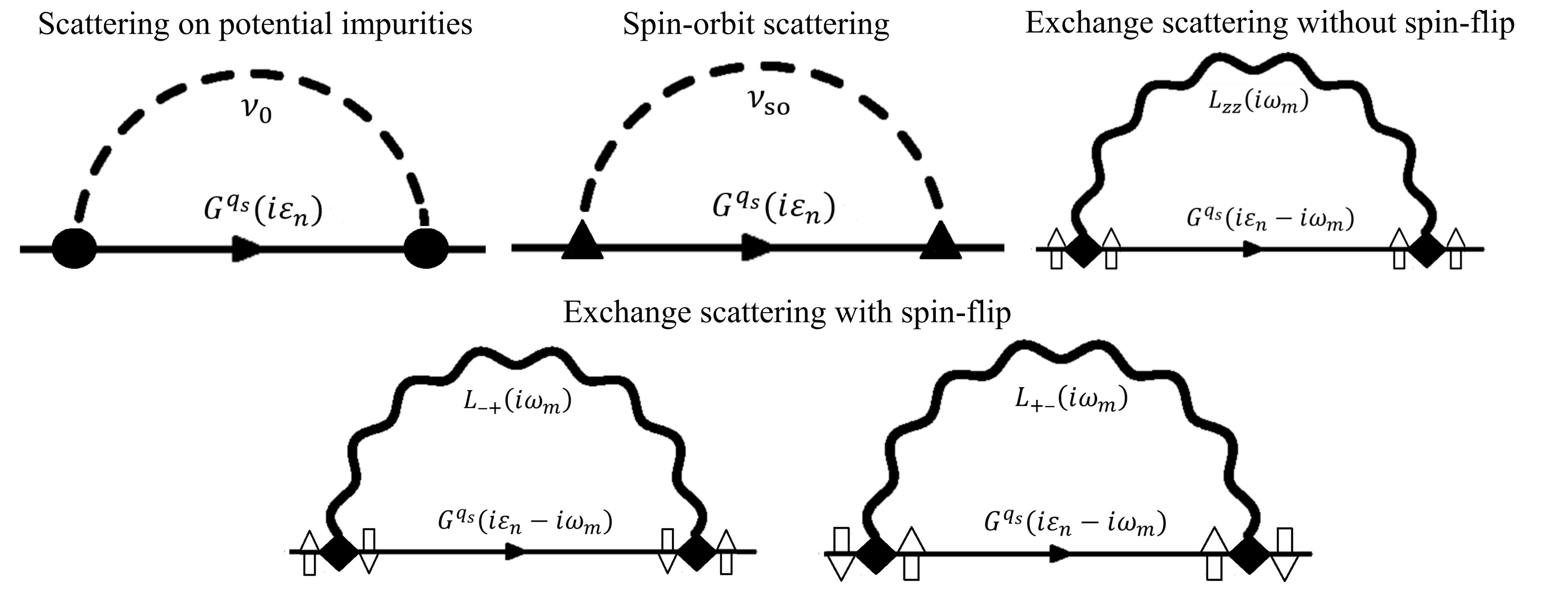


Fig.4: Self-energy diagrams which arise in the Born approximation.

SC enhancement at arbitrary T

Green's function equation taking into account spin polarization:

$$\frac{\varepsilon_{n0}}{\Delta_0} = u\left(1 - \underbrace{\frac{\nu_z}{\Delta_0}\frac{1}{\sqrt{1+u^2}}}_{\text{ES without spin-flip}} - \underbrace{\frac{\gamma_\parallel}{\Delta_0}\frac{1}{\sqrt{1+u^2}}}_{\text{OE +PE}}\right) - \underbrace{\frac{\nu_\perp T}{2\Delta_0}\left[\sum_{\omega_m} \frac{2\omega_s}{\omega_s^2 + \omega_m^2}\frac{u}{\sqrt{1+\tilde{u}^2}} + \sum_{\omega_m} \frac{2\omega_s}{\omega_s^2 + \omega_m^2}\frac{\tilde{u}}{\sqrt{1+\tilde{u}^2}}\right]}_{\text{ES with spin-flip}} \qquad \tilde{u} = u(\varepsilon_{n0} - i\omega_m)$$

Analysis in limiting cases:

1) KF theory at $\Delta_0 \to 0$ ($u \to \infty$) where $C_0 = u^{-1}$

2) AG-like Green's function equation at:

- $\omega_s = 0$: $\dfrac{\varepsilon_{n0}}{\Delta_0} = u_s\left(1 - \dfrac{\boldsymbol{\nu_s}}{\Delta_0}\dfrac{1}{\sqrt{1+u_s^2}} - \dfrac{\gamma}{\Delta_0}\dfrac{1}{\sqrt{1+u_s^2}}\right)$
- $\omega_s = \infty$: $\dfrac{\varepsilon_{n0}}{\Delta_0} = u_s\left(1 - \dfrac{\nu_\infty}{\Delta_0}\dfrac{1}{\sqrt{1+u_s^2}} - \dfrac{\gamma}{\Delta_0}\dfrac{1}{\sqrt{1+u_s^2}}\right)$

SC enhancement due to $\nu_\infty < \nu_s$

→ Possibility for the transition from the gapless to gapfull state.

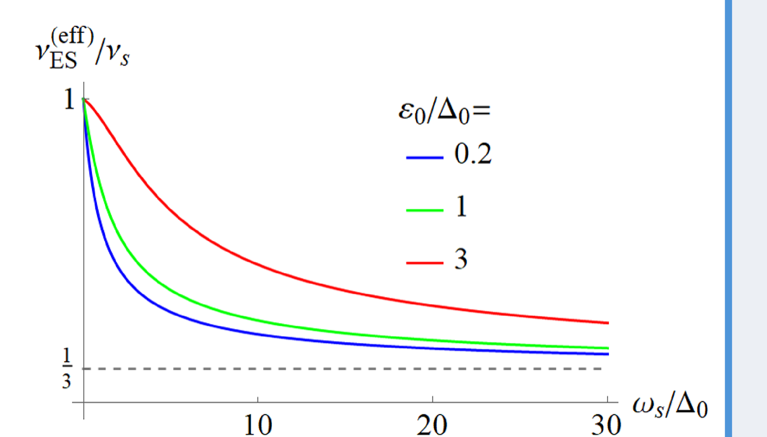
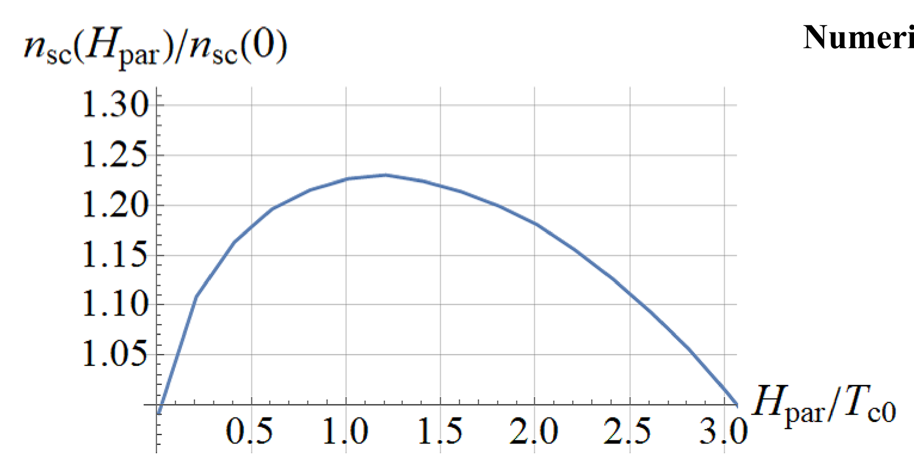


Fig.5: $\nu_{ES}^{(\text{eff})}(\omega_s)$ for $S = 1/2$ at $T = 0$ and different energies ε_{n0} in $\nu_s \ll T_{c0}$ limit.



Numerical results

$$\ln\frac{\Delta_0}{\Delta_{00}} = \frac{\pi T}{\Delta_0}\sum_{\varepsilon_{n0}}\left(\frac{1}{\sqrt{1+u_s^2}} - \frac{\Delta_0}{\varepsilon_{n0}}\right)$$

$$\frac{n_{sc}}{n} = \pi T \sum_{\varepsilon_{n0}} \frac{1}{\nu_0(1+u_s^2)}$$

Fig.6: $n_{sc}(H_{par})$ dependence at $\nu_s = 0.7\,T_{c0}$, $\nu_{so} = 10^3 T_{c0}$, $\nu_0 = 10^4 T_{c0}$, $p_F d = 30$, $p_F l = 10$, $\zeta = 5$

h_{c2} enhancement

Previously, only parallel to film surface field was considered. We add **perpendicular** field component and investigate $h_{c2}(h_\parallel)$ enhancement.

Modification of the Cooperon equation:

$$\left(|\varepsilon_{n0}| + \nu_{ES}^{(\text{eff})}(\varepsilon_{n0}) + \frac{1}{2}\left(\hat{L}_0 - \nu_\perp(\varepsilon_{n0})\right) + \frac{3\left(h_\parallel^2 + h_\perp^2\right)}{2\nu_{so}} + \frac{D}{2}\left(-i\frac{\partial}{\partial \boldsymbol{r}} - \frac{2e}{c}\boldsymbol{A}\right)^2\right) C_0(\varepsilon_{n0}, \boldsymbol{r}, \boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

1) Averaging over film thickness (for ⊥ direction).
2) Solving the Schrödinger equation for in-plane (∥) direction

Full pair-breaking parameter

$$\gamma = \gamma_\parallel + \gamma_\perp$$

$$\gamma_\parallel = \frac{3(h_\parallel')^2}{2\nu_{so}} + \frac{2(p_F d)^2 (h_\parallel)^2}{9\nu_0}$$

$$\gamma_\perp = \frac{3(h_\perp')^2}{2\nu_{so}} + \frac{2\,(p_F l) h_\perp}{3}$$

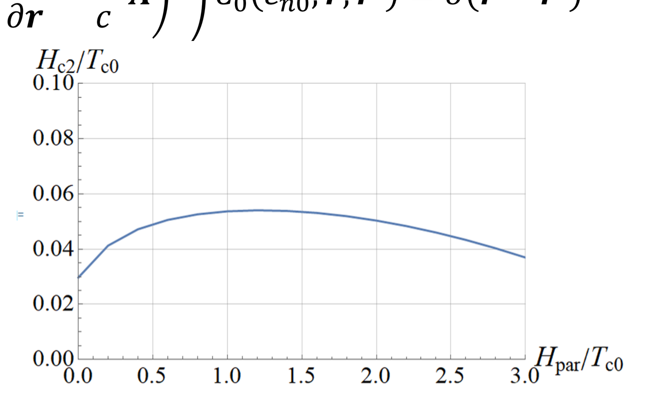


Fig.7: $H_{c2}^{perp}(H_{par})$ dependence at $\nu_s = 0.7\,T_{c0}$, $\nu_{so} = 10^3 T_{c0}$, $\nu_0 = 10^4 T_{c0}$, $p_F d = 30$, $p_F l = 10$, $\zeta = 5$

Conclusions

We investigate the mechanism of superconductivity enhancement due to the polarization of the impurity spins in the presence of external magnetic field using the Gor'kov's diagrammatic technique.

We extend the theory developed in Ref. [1] in two directions:

1. For $h_\perp = 0$ to arbitrary temperatures:
   - Derive the Green's function equation taking into account spin polarization.
   - Demonstrate that at $\omega_s = 0$ and $\omega_s = \infty$ the Green's function equation reproduces the AG theory but with $\nu_{ES}^{(\text{eff})}$ instead of ν_s.
   - Demonstrate n_{sc} enhancement in the presence of the magnetic field.
2. For $h_\perp \neq 0$:
   - Demonstrate H_{c2} enhancement applying the parallel (to the film surface) field.

Literature

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